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Unsteady flow and heat transfer of a generalized Maxwell fluid due to a hyperbolic sine accelerating plate

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ABSTRACT

This paper deals with the unsteady flow and heat transfer of a generalized Maxwell fluid over a moving flat plate with variable temperature and hyperbolic sine velocity. Exact solutions are established for the velocity and temperature fields in terms of discrete Fourier sine transform coupled with Laplace transform for the fractional calculus. Graphs are sketched for values of parameters and associated dynamic characteristics are analyzed.

1. Introduction

In recent years, the advance in technological applications have brought a wide range of rheological complex fluids that are characterized by diverse and significant deviations from simple Newtonian fluids and a considerable attention has been devoted to predict the behavior of rheological fluids [1]. The fractional calculus were found to be quite flexible for describing the rheological and viscoelastic properties of fluids [2–5]. In this paper, we consider the unsteady flow and heat transfer of a generalized Maxwell fluid over a moving flat plate due to a hyperbolic sine velocity. We assume that the fluid has an initial temperature, the temperature of the plate changes with time. The exact solutions and numerical ones are obtained by using the discrete Fourier sine transform coupled with Laplace transform for the fractional calculus [6,7] and the associated velocity field and the temperature field dynamic characteristics are illustrated by figures.

2. Governing equations

The constitutive equations of generalized Maxwell fluid are defined [2–5]

$$\mathbf{T} = -p\mathbf{I} + \mathbf{S}, \quad \mathbf{S} + \lambda(D_t^{\alpha}\mathbf{S} + \mathbf{V}\cdot\nabla\mathbf{S} - \mathbf{L}\mathbf{S} - \mathbf{S}\mathbf{L}^T) = \mu\mathbf{A} \tag{1}$$

$$(1 + \lambda D_t^{\alpha})\tau(y, t) = \mu\partial_y u(y, t) \tag{2}$$

$$\rho\partial_t u(y, t) = \partial_y \tau(y, t). \tag{3}$$

In the absence of a pressure gradient in the x-direction, the equation of motion is written as

$$(1 + \lambda D_t^{\alpha})\frac{\partial u(y, t)}{\partial t} = \upsilon\frac{\partial^2 u(y, t)}{\partial y^2}. \tag{4}$$

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One-dimensional unsteady heat conduction equation in generalized Maxwell fluid is written as

$$\rho c \frac{D^a T}{\partial t^a} = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y}\right)^2, \quad 0 \le \alpha \le 1 \tag{5}$$

where D_t^α is fractional differentiation operators of order α $(0 \le \alpha < 1)$ with respect to t based on Riemann–Liouville's definition [3–7]. For the physical meaning of present fractional differential equations, see [8] and reference therein.

3. Formulation of the problem and solutions

We consider an incompressible generalized Maxwell fluid over an infinite flat plate. Initially, the fluid is at rest, and at the time $t = 0^+$, the infinite plate begins to move in the speed of $\sinh\ at = \frac{1}{2}(e^{at} - e^{-at})$

$$u(y,0) = \frac{\partial u(y,0)}{\partial t} = 0, \quad y > 0, \quad u(0,t) = \sinh\ at, \quad t \ge 0 \tag{6}$$

where a is a constant. In terms of Laplace transform, Fourier sine transform and their inverse transform, we get

$$\begin{aligned} u(y,t) \;=\; & \frac{\upsilon}{\pi\lambda}\int_0^\infty \xi \sin(y\xi) \int_0^t e^{a(t-s)} \sum_{k=0}^\infty \sum_{j=0}^\infty \frac{(-1)^k}{\lambda^k} \frac{(k+1)_j t^{\alpha k+\alpha j+\alpha+j}}{\Gamma(1+j)\Gamma[\alpha k+\alpha j+\alpha+j+1]} \left(-\frac{\upsilon\xi^2}{\lambda}\right)^j \mathrm{d}s\mathrm{d}\xi \\ & - \frac{\upsilon}{\pi\lambda}\int_0^\infty \xi \sin(y\xi) \int_0^t e^{-a(t-s)} \sum_{k=0}^\infty \sum_{j=0}^\infty \frac{(-1)^k}{\lambda^k} \frac{(k+1)_j t^{\alpha k+\alpha j+\alpha+j}}{\Gamma(1+j)\Gamma[\alpha k+\alpha j+\alpha+j+1]} \left(-\frac{\upsilon\xi^2}{\lambda}\right)^j \mathrm{d}s\mathrm{d}\xi \end{aligned} \tag{7}$$

$$\bar{\tau}(y,s) = \frac{\mu}{\lambda s^\alpha + 1} \frac{\partial \bar{u}(y,s)}{\partial y} \tag{8}$$

$$\begin{aligned} \tau(y,t) \;=\; & \frac{\upsilon\mu}{\pi\lambda^2}\int_0^\infty \xi^2 \cos(y\xi) \sum_{k=0}^\infty \sum_{j=0}^\infty \frac{(-1)^k}{\lambda^k} \left(-\frac{\upsilon\xi^2}{\lambda}\right)^j \frac{(k+1)_j}{\Gamma(1+j)} \int_0^t \int_0^\sigma \frac{e^{a(\sigma-s)} s^{ak+aj+a+j}}{\Gamma(ak+aj+a+j+1)} R_{a,0} \\ & \times \left(\frac{-1}{\lambda}, 0, t-\sigma\right) \mathrm{d}s\mathrm{d}\sigma\mathrm{d}\xi - \frac{\upsilon\mu}{\pi\lambda^2}\int_0^\infty \xi^2 \cos(y\xi) \sum_{k=0}^\infty \sum_{j=0}^\infty \frac{(-1)^k}{\lambda^k} \left(-\frac{\upsilon\xi^2}{\lambda}\right)^j \frac{(k+1)_j}{\Gamma(1+j)} \\ & \times \int_0^t \int_0^\sigma \frac{e^{-a(\sigma-q)} s^{ak+aj+a+j}}{\Gamma(ak+aj+a+j+1)} R_{a,0}\left(\frac{-1}{\lambda}, 0, t-\sigma\right) \mathrm{d}s\mathrm{d}\sigma\mathrm{d}\xi \qquad . \end{aligned} \tag{9}$$

Assume that the initial temperature of the fluid is T_0, and the temperature of the plate is $T_1 + T_2 \sin(\omega t)$, we obtain

$$\begin{aligned} T(y,t) \;=\; & \sqrt{\frac{2}{\pi}}\int_0^\infty \sin(qy)\Phi(q) \sum_{n=0}^\infty \frac{(-aq^2t^\alpha)^n}{\Gamma(\alpha n+1)} \mathrm{d}q + \sqrt{\frac{2}{\pi}}\int_0^\infty \sin(qy) \int_0^t F(q,\tau)(t-\tau)^{\alpha-1} \\ & \times \sum_{n=0}^\infty \frac{[-aq^2(t-\tau)^\alpha]^n}{\Gamma(\alpha n+\alpha)} \mathrm{d}\tau\mathrm{d}q + \frac{2aT_1}{\pi}\int_0^\infty q\sin(qy) \int_0^t (t-\tau)^{\alpha-1} \sum_{n=0}^\infty \frac{[-aq^2(t-\tau)^\alpha]^n}{\Gamma(\alpha n+\alpha)} \mathrm{d}\tau\mathrm{d}q \\ & + \frac{2aT_2}{\pi}\int_0^\infty q\sin(qy) \int_0^t \sin(\omega\tau)(t-\tau)^{\alpha-1} \sum_{n=0}^\infty \frac{[-aq^2(t-\tau)^\alpha]^n}{\Gamma(\alpha n+\alpha)} \mathrm{d}\tau\mathrm{d}q \end{aligned} \tag{10}$$

$$a = \frac{k}{\rho c}, \qquad f(y,t) = \frac{\mu}{\rho c}\left(\frac{\partial u}{\partial y}\right)^2, \qquad F(q,t) = \sqrt{\frac{2}{\pi}}\int_0^\infty \sin(yq) f(y,t)\mathrm{d}y.$$

4. Results and analysis

In order to investigate the characteristics of the solutions, some graphs are sketched for values of parameters. Figs. 1 and 2 describe the distributions of velocity and shear stress fields. These charts indicate that the velocity and shear stress showed significant changes with time t, and also showed slight changes with values of fractional order derivative α. The velocity and the shear stress decrease with increase in time t. Fig. 3 shows the distribution of temperature field, these figures also indicated that the temperature field showed significant changes with time t and a slight change with different values of fractional order derivative α. The greater the values of t, the rapid the temperature decays.

5. Conclusions

This paper presented a research for unsteady flow and heat transfer of a generalized Maxwell fluid over a moving flat plate with variable temperature and hyperbolic sine velocity. The exact solutions and numerical ones are obtained by using the discrete Fourier sine transform and Laplace transform and expressed as double integrals of double series and illustrated by figures. The associate velocity field and the temperature field dynamic characteristics are analyzed.

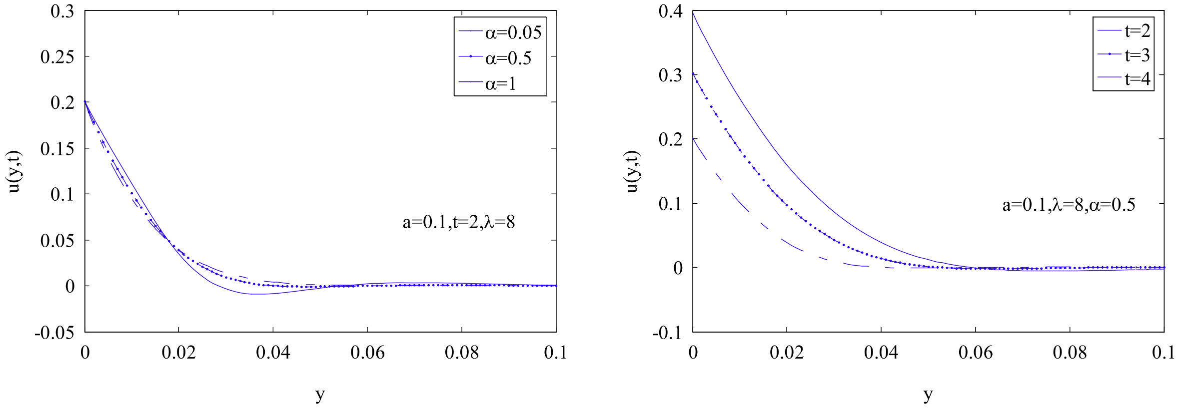

Fig. 1. Comparison of velocity fields for different values of α and t.

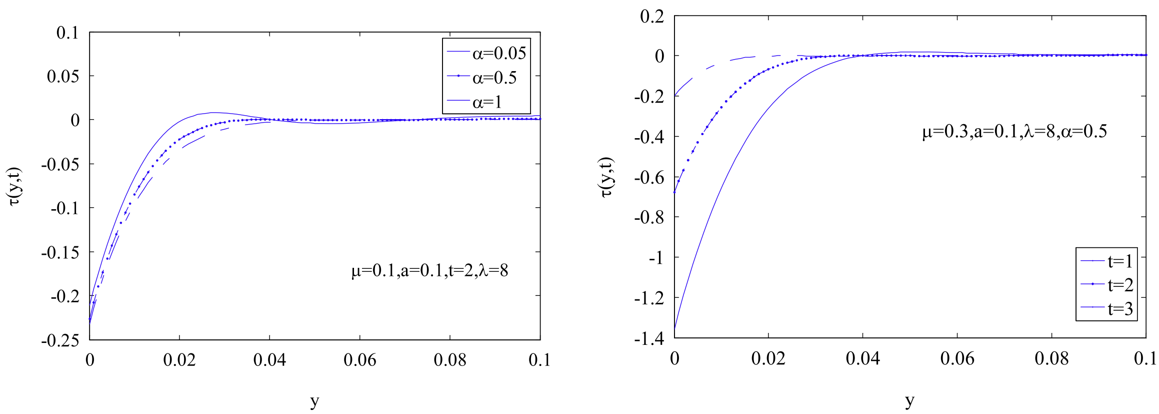

Fig. 2. Comparison of shear stress fields for different values of α and t.

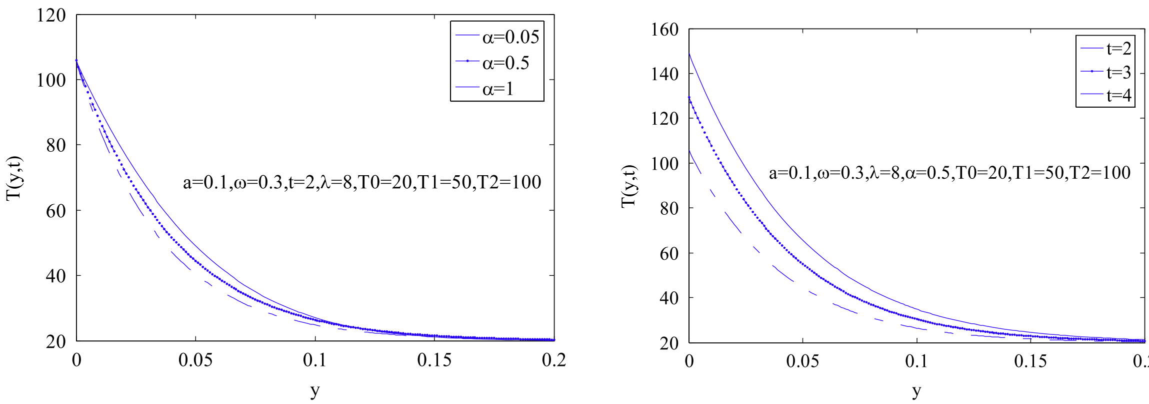

Fig. 3. Comparison of temperature fields for different values of α and t.

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